31.53. Model: Assume that the connecting wires are ideal, but the battery is not. The battery has internal resistance. Also assume that the ammeter does not have any resistance.
Visualize: Please refer to Figure P31.53.
Solve: When the switch is open,

$$\mathcal{E} - Ir - I(5.0\ \Omega) = 0\text{ V} \Rightarrow \mathcal{E} = (1.636\text{ A})(r + 5.0\ \Omega)$$

where we applied Kirchhoff's loop law, starting from the lower left corner. When the switch is closed, the current I comes out of the battery and splits at the junction. The current $I' = 1.565$ A flows through the 5.0 Ω resistor and the rest $(I - I')$ flows through the 10.0 Ω resistor. Because the potential differences across the two resistors are equal,

$$I'(5.0\ \Omega) = (I - I')\,(10.0\ \Omega) \Rightarrow (1.565\text{ A})(5.0\ \Omega) = (I - 1.565\text{ A})(10.0\ \Omega) \Rightarrow I = 2.348\text{ A}$$

Applying Kirchhoff's loop law to the left loop of the closed circuit,

$$\mathcal{E} - Ir - I'(5.0\ \Omega) = 0\text{ V} \Rightarrow \mathcal{E} = (2.348\text{ A})r + (1.565\text{ A})(5.0\ \Omega) = (2.348\text{ A})r + 7.825\text{ V}$$

Combining this equation for $\mathcal{E}$ with the equation obtained from the circuit when the switch was open,

$$(2.348\text{ A})r + 7.825\text{ V} = (1.636\text{ A})r + 8.18\text{ V} \Rightarrow (0.712\text{ A})r = 0.355\text{ V} \Rightarrow r = 0.50\ \Omega$$

We also have $\mathcal{E} = (1.636\text{ A})(0.50\ \Omega + 5.0\ \Omega) = 9.0$ V.